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OPTIMUM LAUNCH VEHICLE SIZING.(U)

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(6) # Optimum Launch Vehicle Sizing.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A FORTRAN program that computes the optimum sizing parameters of a multi-stage rocket in powered ascent has been developed. The computer program is based upon an optimization procedure, due to Ruppe, which maximized the payload weight to total launch vehicle weight. Based upon a simplified aerodynamic model, the optimization procedure is convenient to use and provides useful approximate values of launch vehicle sizing parameters for launch vehicle of n stages, where n = 0, 1, 2... A numerical example is given to illustrate the utility of the FORTRAN program.

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CONTENTS

OPTIMUM LAUNCH VEHICLE SIZING

I. INTRODUCTION

In launch vehicle sizing, it is often necessary to determine the relationships between the parameters of vehicle size and the parameters of mission requirements for multi-stage rockets in powered ascent. Relationships of this type are generally arrived at as a result of an optimization procedure based upon some reasonable criterion, such as maximizing the payload weight to launch weight ratio, or minimizing the total launch vehicle cost. In a particular situation, the mission may be to attain certain launch altitude and velocity. Then, if a set of preliminary design parameters are also known (such as the number of stages, the specific impulse of the stages and the structure ratios of each stage), the output of the optimization program will provide sizing parameters for each of the launch vehicle stages.

The major forces experienced by a multi-stage rocket in powered ascent are: a thrust force from the rocket motors, aerodynamic forces of drag and lift and a gravitational force of attraction towards the earth. Drag and lift forces are nonlinear functions of vehicle velocity, while the gravitation force depends upon vehicle altitude and is also a nonlinear function of altitude. Because of these facts, a general analytical optimization procedure for launch vehicle sizing taking into account the aerodynamic forces and the gravitational force, remains an unsolved problem. For cases where precise values of vehicle sizing are required, designers have resorted to large scale numerical computation programs which examines the value of each parameter individually while fixing the value of remaining parameters. While this numerical method does provide more accurate values, compared to those from an optimization procedure to be discussed, it is time consuming, tedious and does not provide a picture of the relationships between the parameters of launch vehicle size and the parameters of mission requirement. In addition, in a preliminary vehicle design effort an optimization approach based upon a simplified model often provides results of sufficient accuracy.

In this report, a brief description of an optimization procedure for launch vehicle sizing for multi-stage rockets in powered flight based upon a

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simplified model will be presented. This will be followed by an outline of the analytical relationships for a Fortran program which can be used to compute preliminary launch vehicle parameters.

II. OPTIMIZATION OF PAYLOAD WEIGHT TO TOTAL LAUNCH WEIGHT RATIO

In this section, we briefly describe an optimization procedure due to Ruppe[1] which can be used to compute the launch vehicle parameters for the case when the payload-to-launch weight ratio is optimized (maximized). The analytical procedure is developed by assuming the step rocket is in vertical ascent, in vacuum, under constant thrust per stage. Under these assumptions, the gravitational and aerodynamic forces are neglected so that the assumed model is not realistic. However, for initial vehicle sizing, the results obtained do provide useful information.

The optimum weight distribution for a multi-stage or step rocket in vertical ascent, under constant thrust (per stage), in vacuum will be outlined. First the following definitions are required:

$w_o(i)$ = weight of vehicle at ignition of step No. i
$w_c(i)$ = weight of vehicle at cutoff of step No. i
$w_1(i)$ = payload weight of step No. i
= weight of vehicle at ignition of step No. i + 1
$w_s(i)$ = weight of structure of step No. i

Figure 1 shows a 3-stage multi-stage rocket.

In a multi-stage rocket, the first stage accelerates to a burnout velocity V_1, at which time the exhausted first stage structure is ejected and the second stage ignition starts. The second stage, which starts with initial velocity V_1, accelerates the vehicle by a velocity increment of V_2, so that at the end of stage two burn, the vehicle velocity is $V_1 + V_2$. This process continues for additional stages until the final burnout velocity of the payload is given by $V_1 + V_2 + \ldots V_N$, where N is total number of stages. To continue, the following definition will also be required:

MASS ratio of step No. i

$$r_i = \frac{W_o(i)}{W_c(i)} \qquad (1)$$

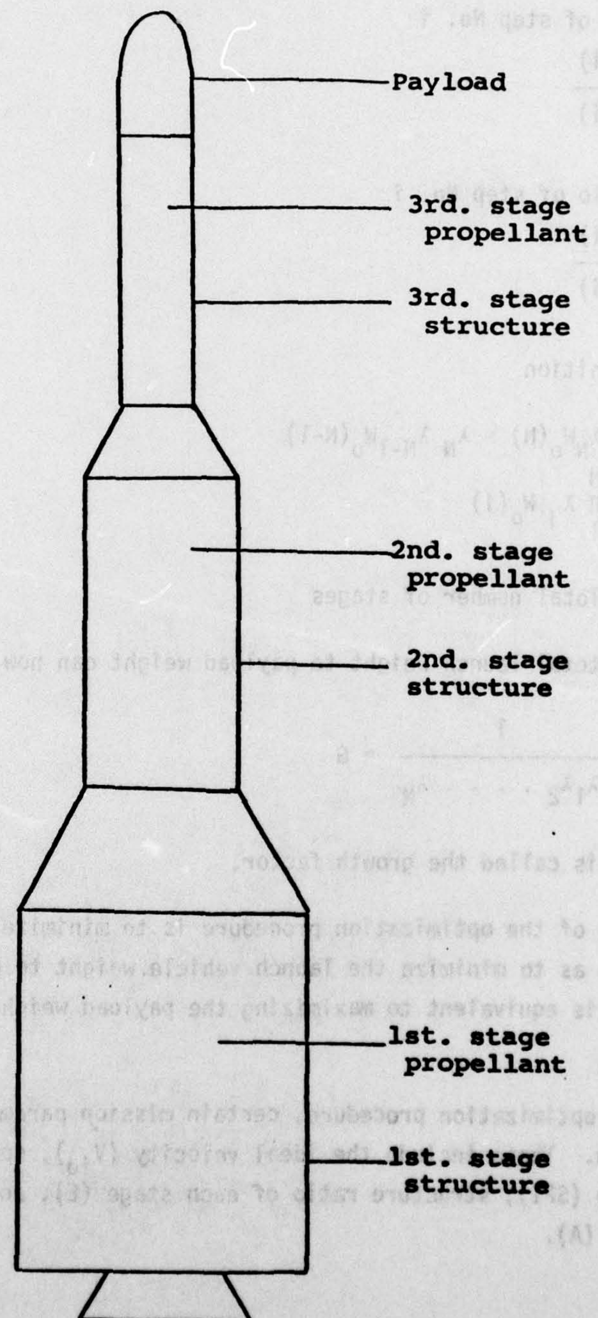

Fig. 1 - 3 Stage Rocket

Payload ratio of step No. i

$$\lambda_i = \frac{W_1(i)}{W_o(i)} \tag{2}$$

Structure ratio of step No. i

$$E_i = \frac{W_s(i)}{W_o(i)} \tag{3}$$

From the definition

$$\begin{aligned} W_1(N) &= \lambda_N W_o(N) = \lambda_N \lambda_{N-1} W_o(N-1) \\ &= \prod_1^N \lambda_i W_o(i) \end{aligned} \tag{4}$$

N = Total number of stages

The ratio of total launch weight to payload weight can now be written as

$$\frac{W_o(i)}{W_i(N)} = \frac{1}{\lambda_1 \lambda_2 \cdot \cdot \cdot \cdot \lambda_N} = G \tag{5}$$

Where G is called the growth factor.

The objective of the optimization procedure is to minimize the growth factor, G, so as to minimize the launch vehicle weight to payload weight ratio. This is equivalent to maximizing the payload weight to launch weight ratio.

In the Ruppe optimization procedure, certain mission parameters are assumed known. These include the ideal velocity (V_{id}), specific impulse of each stage (SPI), structure ratio of each stage (E), and the initial load factors (A).

The ideal velocity is defined by:

$$V_{id} = V_m + V_w \pm V_{er} + V_{sl} + V_g + V_a \tag{6}$$

where V_m is the desired mission velocity, V_w is the velocity component allocated for launch windows capability, V_{er} is the velocity component due to earth rotation, V_{sl} is the velocity component for system losses (such as variations in motor efficiency), V_g is velocity component due to gravity loss and V_a is velocity component due to aerodynamic effects, such as drag and lift.

Then, the optimization procedure first solves for the unknown, λ_N from

$$\bar{R} = \frac{\prod_{1}^{N}\left(1 - \frac{C_N}{C_i}\frac{\lambda_N}{E_N + \lambda_N}\right)^{C_i/\bar{c}}}{\prod_{1}^{N} E_i^{\;C_i/\bar{c}}} \tag{7}$$

where $\bar{R}$ is computed from

$$\bar{R} = \exp\left(V_{id}/\bar{c}\right) \tag{8}$$

With $\bar{C}$ being the average exhaust velocity of the stages given by

$$\bar{C} = \frac{1}{N}\sum_{1}^{N} C_j, \tag{9}$$

where C_j is the exhaust velocity of gas of jth. stage. Under our assumption of vacuum and constant gravitational acceleration,

$$C_j = SPI_{(j)} g_o, \tag{10}$$

where $SPI_{(j)}$ is the specific impulse of jth. stage and g_o is the acceleration of gravity at earth's surface.

Since in Eq. 7, λ_N is the only unknown, the value of λ_N must be solved by an iterative procedure. Once this is accomplished, λ_i, $i = 1, 2 \ldots N-1$, can be obtained from

$$\lambda_i = \frac{E_i}{1 - \frac{C_N}{C_i}\left(\frac{\lambda_N}{E_N + \lambda_N}\right)} - E_i. \tag{11}$$

With λ_i now known, the calculation can proceed to determine the parameters of vehicle sizing in the following:

Stage Weight (lbs)

$$W_o(1) = G \cdot PAY(N) \tag{12}$$

Fuel Weight (lbs)

$$W_f(i) = W_o(i)\left(\frac{r_i - 1}{r_i}\right)$$

$$r_i = \frac{1}{\lambda_i + E_i} \tag{13}$$

Structure Weight (lbs)

$$W_s(i) = W_o(i) \cdot E_i \tag{14}$$

Payload Weight of ith stage (lbs)

$$PAY(i) = W_o(i) - W_f(i) - W_s(i) \tag{15}$$

Exhaust Velocities (kilometer/sec)

$$C_i = SPI_{(i)} \cdot g_o \tag{16}$$

Thrust (lbs)

$$T(i) = A(i)W_o(i) \qquad (17)$$

Burn time (sec)

$$T_b(i) = \frac{W_f(i)}{T(i)} \cdot SPI_{(i)} \qquad (18)$$

III. FORTRAN PROGRAM

Rally I (Section V) is a Fortran program which computes the launch parameters of a step rocket for the case when the payload weight to launch vehicle weight is maximized. The program is based upon the model which assumes the step rocket is in vertical ascent, in vacuum, and under constant thrust per stage. Although these assumptions may seem somewhat restrictive, the results obtained do provide first order approximations for launch vehicle design.

A list of the program INPUTS and OUTPUTS will now be provided:

INPUTS

A. Structure ratios, E_i, entered as data in step 200 under EPS for values corresponding to stages 1, 2, 3, . . . N, in order.

B. Stage exhaust velocities (kilometer/sec), entered as DATA in step 200 under C for stages 1, 2, 3, . . . N in order.

C. Specific impulses, entered as DATA in step 210 under SPI for stages 1, 2, 3, . . . N in order.

D. Initial load factors (ratio of thrust to stage weight) entered as DATA in step 220 under AK for stages 1, 2, 3, . . . N in order.

E. Payload weight, entered in step 400 under PAY in lbs.

F. Number of stages, entered in step 500 under N. There is no restriction on the value of N, except the values in the DIMENSION size (steps 100, 110, 120) must be made equal to or greater than N.

G. Ideal velocity, V_{id} (kilometers/sec), entered in step 600 under VID.

OUTPUTS

The OUTPUTS of RALLY 1 are:

A. The growth factor, G

B. Thrust (lbs) per stage

C. Stage weight (lbs) per stage

D. Fuel weight (lbs) per stage

E. Payload weight (lbs) per stage

F. Structure weight (lbs) per stage

G. Specific impulse (seconds) per stage

H. Burn time (seconds) per stage

IV. NUMERICAL EXAMPLES

In this section, a sample calculation will be given to show the use of FORTRAN Program RALLY 1. For convenience, we consider a 2-stage rocket whose mission is to launch a payload into a circular orbit at an altitude of 100 nautical miles. The mission parameters for this example are given in a U. S. Air Force document titled: Space Planner's Guide[2], (SPG), page V-20. In this document, a graphic procedure for vehicle sizing, which maximizes the payload weight to launch vehicle weight, is given. The results of this graphic procedure will be compared to the values obtained from Rally 1.

A. Mission Parameter

The following mission parameters were given in SPG

Orbit	100 Nautical Miles, Circular	
Number of stages	2	
Payload Weight	81,324 lbs	
Ideal velocity	32,480 ft/sec	
	Stage 1	Stage 2
Vacuum specific impulse	395 sec	419 sec
Thrust to weight ratio	1.3	1.0

TABLE 1 - Mission Parameters (SPG)

B. Rally 1 Parameters

In order to use Rally 1, the mission parameters of Table 1 are converted into appropriate units shown in Table 2.

Number of stages	2	
Payload Weight	81,324 lbs	
Ideal Velocity	9.885 km/sec	
	STAGE 1	STAGE 2
Exhaust velocity	3.871 km/sec	4.1062 km/sec
Load factors	1.3	1.0
Structure ratio	.0665	.0965
Specific impulse	395	419

TABLE 2 - Rally 1 Inputs

In Table 2, the ideal velocity was obtained simply by conversion of units into km/sec. The exhaust velocity was obtained by using Eq. 16.

$$C_j = SPI_{(j)} \cdot g_o \tag{18}$$

where c_j is the exhaust velocity of the jth stage, g_o is the acceleration of gravity at earth's surface (km/sec), and $SPI_{(j)}$ is the vacuum specific impulse. The values of "structure ratios", E, are computed from the values of "structure factor" and the values of stage weights shown in Table 3 for the SPG case. Since the definition of "structure factor" is

$$S_f(i) = \frac{W_s(i)}{W_o'(i)} \tag{19}$$

where $S_f(i)$ is the structure factor of ith stage, $W_s(i)$ is the structure weight of ith stage and $W_o'(i)$ is the stage weight of <u>ith stage only</u> (does not include weight of stages i + 1, i + 2, . . . N); the relationship between E(i) and $S_f(i)$ can be determined by noting

PARAMETERS	SPG STAGE 1	SPG Stage 2	RALLY 1 Stage 1	RALLY 1 Stage 2
Stage weight (lbs)	1.854m	386,000	1.875m	360,045
Structure factor	0.0840	.0965		
Structure Ratio			.0665	.0965
Propellant Weight (lbs)	1.340 m	275,000	1.389m	244,343
Load Factor	1.3	1.0	1.3	1.0
Thrust (lbs)	2.41m	286,000	2.44m	360,450
Stage Velocity Increment (ft/sec)	16,050	16,400	17,197	15,285
Growth factor	22.798		23.057	

Table 3 - Comparison of SPG and Rally Results

$$W_o(1) = W_o'(1) + W_o'(2)$$

Since

$$W_o(2) = W_o'(2),\ W_o'(1) = W_o(1) - W_o'(2) \tag{20}$$

The structure factor of Eq. 19 becomes

$$S_f(1) = \frac{W_s(1)}{W_o(1) - W_o'(2)} \tag{21}$$

Dividing by $W_o(1)$, we note the relationship between E and S_f

$$E(1) = S_f(1)\left(1 - \frac{W_o(2)}{W_o(1)}\right) \tag{22}$$

$$E(2) = S_f(2) \tag{23}$$

Thus E (1) and E (2) can be computed from the values of SPG data given in Table 3.

$S_f(1) = .0840$
$S_f(2) = .0965$
$W_o(1) = 1{,}854{,}000$ lbs
$W_o(2) = 386{,}000$ lbs

The computed values of launch vehicle parameters using Fortran Program Rally 1 are shown in Table 4. A summary of the computations are shown in Table 3 for comparison, and reasonable agreement between the SPG values and Rally 1 values can be seen for the 2-stage rocket under consideration.

```
.....CBAR......RBAR........X.........LAMN
...3.98860...11.92114....0.70042....0.22562


LAMBDA NUMBER 1 IS ....0.19223
LAMBDA NUMBER 2 IS ....0.22562


LOAD RATIO NUMBER 1 IS ....3.86501
LOAD RATIO NUMBER 2 IS ....3.10446


DELTA V NUMBER 1 IS ....5.23345
DELTA V NUMBER 2 IS ....4.65166


GROWTH FACTOR IS ...23.05693


PARAMETERS FOR STAGE 1 FOLLOW

THE THRUST IS  2437606.7 POUNDS
THE STAGE WEIGHT IS  1875082.1 POUNDS
THE FUEL WEIGHT IS  1389938.6 POUNDS
THE PAYLOAD WEIGHT IS   360450.5 POUNDS
THE STRUCTURE WEIGHT IS   124693.0 POUNDS
THE SPECIFIC IMPULSE IS  395.00000 SECONDS
THE BURN TIME IS  225.23147 SECONDS


PARAMETERS FOR STAGE 2 FOLLOW

THE THRUST IS   360450.5 POUNDS
THE STAGE WEIGHT IS   360450.5 POUNDS
THE FUEL WEIGHT IS   244343.1 POUNDS
THE PAYLOAD WEIGHT IS    81324.0 POUNDS
THE STRUCTURE WEIGHT IS    34783.5 POUNDS
THE SPECIFIC IMPULSE IS  419.00000 SECONDS
THE BURN TIME IS  284.03271 SECONDS
```

TABLE 4 - Rally 1 OUTPUTS

V. PROGRAM LISTING

```
.TYPE RALLY1.FOR
            DIMENSION EPS(5),C(5),ALAM(5),RATIO(5),DELV(5)
            DIMENSION WF(5),WS(5),SW(5),SPI(5),THR(5),BT(5)
            DIMENSION AK(5)
            DATA EPS/.0665,.0965,0.,0.,0./,C/3.871,4.1062,0.,0.,0./
            DATA SPI/395.,419.,0.,0.,0./
            DATA AK/1.3,1.,0.,0.,0./
            E=2.7182818
            PAY=81324.
            N=2
            VID=9.885
            TYPE 9
            TYPE 3,N
      3     FORMAT(' THE NUMBER OF ROCKET STAGES IS ',I1)
            TYPE 4,VID
      4     FORMAT(' THE IDEAL VELOCITY IS ',F5.1,' KM PER SEC.')
            TYPE 9
            CT=0.0
            DO 10 I=1,N
            CT=CT+C(I)
     10     CONTINUE
            CBAR=CT/N
            RBAR=E**(VID/CBAR)
            ANUM=1.0
            DENOM=1.0
            TRX=0.1
            K=1
     40     DO 20 I=1,N
            ANUM=((1.-(C(N)/C(I))*TRX)**(C(I)/CBAR))*ANUM
            DENOM=EPS(I)**(C(I)/CBAR)*DENOM
     20     CONTINUE
            VAL=ANUM/DENOM
            IF (VAL .LT. RBAR) GO TO 30
            IF (K .EQ. 1) TRX=TRX+0.1
            IF (K .EQ. 2) TRX=TRX+0.01
            IF (K .EQ. 3) TRX=TRX+0.001
            IF (K .EQ. 4) TRX=TRX+0.0001
            IF (K .EQ. 5) TRX=TRX+0.00001
            IF (K .GT. 5) GO TO 32
            ANUM=1.0
            DENOM=1.0
            GO TO 40
     30     IF (K .EQ. 1) TRX=TRX-0.1
            IF (K .EQ. 2) TRX=TRX-0.01
            IF (K .EQ. 3) TRX=TRX-0.001
            IF (K .EQ. 4) TRX=TRX-0.0001
            IF (K .EQ. 5) TRX=TRX-0.00001
            K=K+1
            ANUM=1.0
            DENOM=1.0
            GO TO 40
     32     ALAMN=TRX*EPS(N)/(1.-TRX)
            TYPE 2
      2     FORMAT('    CBAR      RBAR        X           LAMN')
            TYPE 1,CBAR,RBAR,TRX,ALAMN
      1     FORMAT(4(1X,F10.5))
            TYPE 9
```

```
04600            DO 50 I=1,N
04700            ALAM(I)=(EPS(I)/(1.-(C(N)/C(I))*(ALAMN/((EPS(N)+ALAMN))))-EPS
>                1)
04800            TYPE 5,I,ALAM(I)
04900   5        FORMAT(' LAMBDA NUMBER ',I1,' IS ',F10.5)
05000   50       CONTINUE
05100            TYPE 9
05200            DO 60 I=1,N
05300            RATIO(I)=1./(ALAM(I)+EPS(I))
05400            TYPE 6,I,RATIO(I)
05500   6        FORMAT(' LOAD RATIO NUMBER ',I1,' IS ',F10.5)
05600   60       CONTINUE
05700            TYPE 9
05800            DO 70 I=1,N
05900            DELV(I)=C(I)*ALOG(RATIO(I))
06000            TYPE 7,I,DELV(I)
06100   7        FORMAT(' DELTA V NUMBER ',I1,' IS ',F10.5)
06200   70       CONTINUE
06300            TYPE 9
06400            ALAMT=1.0
06500            DO 80 I=1,N
06600            ALAMT=ALAMT*ALAM(I)
06700   80       CONTINUE
06800            G=1./ALAMT
06900            ALW=G*PAY
07000            TYPE 8,G
07100   8        FORMAT(' GROWTH FACTOR IS ',F10.5)
07200   9        FORMAT(//)
07300            SW(1)=ALW
07400            DO 90 I=1,N
07500            WF(I)=SW(I)*((RATIO(I)-1)/RATIO(I))
07600            WS(I)=SW(I)*EPS(I)
07700            SW(I+1)=SW(I)-(WF(I)+WS(I))
07800            THR(I)=AK(I)*SW(I)
07900            BT(I)=WF(I)/THR(I)*SPI(I)
08000   90       CONTINUE
08100            TYPE 9
08200            DO 100 I=1,N
08300            TYPE 140,I
08400   140      FORMAT(' PARAMETERS FOR STAGE ',I1,' FOLLOW'/)
08500            TYPE 150,THR(I)
08600            TYPE 160,SW(I)
08700            TYPE 170,WF(I)
08800            TYPE 180,SW(I+1)
08900            TYPE 190,WS(I)
09000            TYPE 200,SPI(I)
09100            TYPE 210,BT(I)
09110            TYPE 9
09200   150      FORMAT(' THE THRUST IS ',F10.1,' POUNDS')
09300   160      FORMAT(' THE STAGE WEIGHT IS ',F10.1,' POUNDS')
09400   170      FORMAT(' THE FUEL WEIGHT IS ',F10.1,' POUNDS')
09500   180      FORMAT(' THE PAYLOAD WEIGHT IS ',F10.1,' POUNDS')
09600   190      FORMAT(' THE STRUCTURE WEIGHT IS ',F10.1,' POUNDS')
09700   200      FORMAT(' THE SPECIFIC IMPULSE IS ',F10.5,' SECONDS')
09800   210      FORMAT(' THE BURN TIME IS ',F10.5,' SECONDS')
09900   100      CONTINUE
10000            END
```

VI. ACKNOWLEDGEMENTS

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VII. LIST OF REFERENCES

1. H. O. Ruppe, Introduction to Astronautics, Vol. 1, Academic Press, New York, 1966.

2. Space Planner's Guide, United States Air Force, Air Force Systems Command, July 1965.